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RESEARCH ARTICLE

OSCILLATORY FLOW OF COUPLE STRESS FLUID WITH EFFECT OF HALL CURRENT IN AN MULTI-STENOSED ARTERY

[1]Nirmala P Ratchagar and [2]Subasri S

[1,2] Department of Mathematics, Annamalai University, Annamalainagar,Chidambaram– 608002, Tamilnadu, India
Email: nirmalapasala@yahoo.co.in , subasriselvendiran2194@gmail.com

Abstract

In our model, we have considered oscillatory blood flow an impact of Hall current on couple stress blood fluid in multi-stenosis artery. The governing equations are solved to analytical solution. Velocity, temperature, concentration, shear stress and rate of fluid flow are solved by using purely perturbation method. Numerical results are calculated through various parameters depicts the plots.

Key words: Hall current, Oscillatory flow, MHD, Couple stress fluid, Multi- Stenosis, Porous medium

Introduction

The flow of blood through an multi-stenosed artery connected with heart pumping movement of the heart under normal conditions. This pumping fluid of the heart increases due to the pressure gradient which produces an oscillatory blood fluid of the vessels. (Halder, 1987) examined Newtonian fluid as oscillatory blood flow of mild stenosis artery with pressure gradient of harmonic function. Newtonian fluid is assumed to be oscillatory MHD flow of through overlapping stenosis with the effect of blood, solution is analytical and the numerical results are analyzed .(Kumar *et.al.*, 2009). (Rathod and Shakera tanveer, 2009) studied couple stress blood fluid through porous channel effect of periodic body acceleration with magnetic field. (Gaurav Varshney *et.al.*, 2010) developed unsteady blood fluid in multi-stenosed artery analysis numerical solution. Stenosed artery plays an important role in blood fluid (Siddiqui and Sapna Geeta, 2013) developed a mathematical model to study the pulsatile caterterized in periodic body acceleration. (Shit *et.al.*, 2014) studied the flow of blood magnetic field with variable viscosity.

(Srinivasacharya and Madhava Rao, 2015) studied blood fluid through a bifurcated artery with mild stenosis and heat transfer, with blood as couple stress blood fluid. (Bessonov *et.al.*, 2016) investigated modelling methods for blood as a heterogenous fluid the composed of plasma and blood cells. (Riahi, 2017) examined oscillatory blood flow for two case of nonlinear equation steady and unsteady. (Chitra and Karthikeyan, 2018) evaluated oscillatory blood flow effect of MHD in an inclined tapered artery with porous medium. Multiple stenosed artery through a porous oscillatory non Newtonian blood flow impact of magnetic field developed (Sayntan Sarkar and Rajeev Kumar Khave, 2019).

In the present paper we have considered an unsteady oscillatory flow of couple stress blood fluid in multi-stenosed channel with porous medium. The mathematical formulation is solved analytically using perturbation technique. The numerical values are plotted through graphs for various parameters are discussed.

Mathematical formulation

We consider oscillatory pulsatile couple stress blood fluid effect of Hall current horizontal channel. The multi stenosed artery at the boundary of the upper wall $y^* = R$, $T = T_w$ and lower wall $y^* = 0$, $T = T_0$. The blood fluid has an unidirectional $u(y)$ and it is incompressible, viscous and density of blood to be constant.

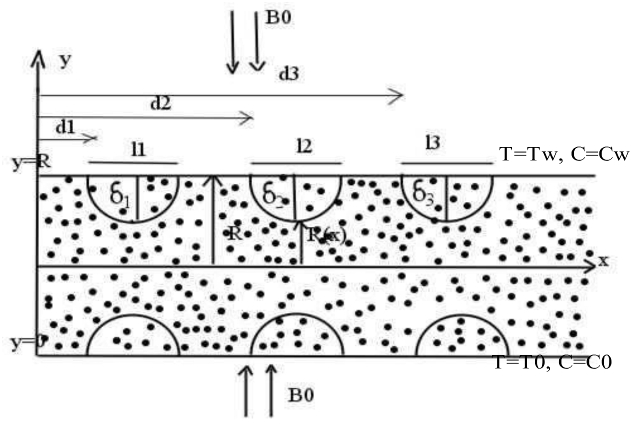


Fig. 1: Physical model.

The approximate fluid motion for blood flow in cartesian coordinate are givenby,

Conservation of continuity:

$$\frac{\partial u^*}{\partial x^*} = 0 \qquad (1)$$

Conservation of momentum:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta\frac{\partial^2 u^*}{\partial y^{*2}} + \lambda\frac{\partial^4 u^*}{\partial y^{*4}} - \frac{\vartheta}{k}u^* - \frac{\sigma B_0^2 u^*}{\rho(1+m^2)} \qquad (2)$$

$$\frac{\partial p^*}{\partial y^*} = 0 \qquad (3)$$

Conservation of energy:

$$\frac{1}{\rho c_p}\,\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^{*2}} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (4)$$

Conservation of concentration:

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_c'(C^* - C_0) \qquad (5)$$

The appropriate boundary condition for the model are given as,

$$\left.\begin{array}{l} u^* = 0, T^* = T_\omega,\ C = C_\omega,\ \dfrac{\partial^2 u^*}{\partial y^{*2}} = 0\ at\ \ y^* = R^* \\ u^* = 0, T^* = T_0,\ C = C_0,\ \dfrac{\partial^2 u^*}{\partial y^{*2}} = 0\ at\ \ y^* = 0 \end{array}\right\} \qquad (6)$$

where,

u^* is the velocity in x direction, g gravitation force, ρ is density of the fluid, p^*

is the pressure, T is temperature of the fluid, k is permeability, K'_C chemical reaction, λ is the slip parameter, ν is kinematic viscosity, μ is the dynamic viscosity, m is Hall current effect, B_0 is external force on magnetic field, c_p is specific heat at constant pressure, P_r is Prandtl number, R_e Reynolds number, T_0 and T_w are temperature(lower and upper wall), C_0 and C_w are concentration (lower and upper wall).

The blood fluid is handle by the throbbing action of the heart which make an unsteady throbbing pressure gradient approximated as (Ogulu, 1993)

$$-\frac{\partial p^*}{\partial x^*} = P_s + \in\ P_0\, Cos(\omega\, t) > 0 \qquad (6)$$

where, $P_s + \in P_0$ the amplitude of the throbbing component giving rise to blood vessel or arterial pressure and heartbeat pressure $w = 2\pi f$ with f, the heart burst frequency.

The geometry of the multi-stenosis in the arterial lumen is described mathematically as,

$$R(x^*) = \begin{cases} R_0 & 0 \le x^* \le {d_1}^* \\ R_0 - \dfrac{{\delta_1}^*}{2}\left(1 + Cos\dfrac{2\pi}{{l_1}^*}\left(x^* - {d_1}^* - \dfrac{{l_1}^*}{2}\right)\right) & {d_1}^* \le x^* \le {d_1}^* + {l_1}^* \\ R_0 & {d_1}^* + {l_1}^* \le x^* \le {d_2}^* \\ R_0 - \dfrac{{\delta_2}^*}{2}\left(1 + Cos\dfrac{2\pi}{{l_2}^*}\left(x^* - {d_2}^* - \dfrac{{l_2}^*}{2}\right)\right) & {d_2}^* \le x^* \le {d_2}^* + {l_2}^* \\ R_0 & {d_2}^* + {l_2}^* \le x^* \le {d_3}^* \\ R_0 - \dfrac{{\delta_3}^*}{2}\left(1 + Cos\dfrac{2\pi}{{l_3}^*}\left(x^* - {d_3}^* - \dfrac{{l_3}^*}{2}\right)\right) & {d_3}^* \le x^* \le {d_3}^* + {l_3}^* \\ R_0 & {d_3}^* + {l_3}^* \le x^* \le l^* \end{cases}$$

where, $R'(x^*)$ is the radius of the artery, R_o is the radius of the normal artery, l_i and δ_i $(i = 1, 2, 3)$ are the length and maximum thickness of three stenosis $(\delta << R_0)$, l is the length of the artery, d is the distance between equispaced point.

Introducing the following non dimensional quantities,

$$x = \frac{x^*}{R_0}, y = \frac{y^*}{R_0}, u = \frac{u^*}{U_0}, R_e = \frac{U_0\, R_0}{\vartheta}, k = \frac{k^*}{R_0^2\,\rho}, t^* = \frac{U_0\, t^*}{R_0}, p^* = \frac{R_0 p^*}{\rho\,\vartheta U_0}, M = \frac{\sigma B_0^2\, R_0^2}{\rho\,\vartheta}$$

$$\theta = \frac{T - T_0}{T_w - T_0}, P_r = \frac{\rho\, c_p}{k}, E_c = \frac{\vartheta_0^2}{C_p(T_1 - T_0)}, l = \frac{l^*}{R_0}, d = \frac{d^*}{R_0}, \delta = \frac{\delta^*}{R_0}, R = \frac{R^*}{R_0}, \alpha^2 = \frac{R_0^2 \omega}{\vartheta},$$

$$l^2 = \frac{\lambda}{\mu}, S_c = \frac{\vartheta}{D} \qquad (7)$$

By using above non dimensional quantities, the equations (1)-(5) and theboundary conditions in (6) reduce to,

$$\frac{\alpha^2 a^2}{2\pi} \frac{\partial u}{\partial t} = -Re\, a^2 \frac{\partial p}{\partial x} + a^2 \left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial^4 u}{\partial y^4} - \left(\frac{M^2}{1 + m^2} + \frac{1}{K}\right) a^2 u \qquad (9)$$

$$\frac{\partial p}{\partial y} = 0 \qquad (10)$$

$$\frac{\alpha^2}{2\pi} \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + E_c \left(\frac{\partial u}{\partial y}\right)^2 \qquad (11)$$

$$\frac{\alpha^2 S_c}{2\pi} \frac{\partial \varphi}{\partial t} = \frac{\partial^2 \varphi}{\partial y^2} - K_c \varphi \qquad (12)$$

with the boundary conditions,

$$\left.\begin{aligned} u = 0, \theta = 1,\ \varphi = 1,\ \frac{\partial^2 u}{\partial y^2} = 0\ at\ \ y = R \\ u = 0, \theta = 0,\ \varphi = 0,\ \frac{\partial^2 u}{\partial y^2} = 0\ at\ \ y = 0 \end{aligned}\right\} \qquad (13)$$

where P_r is Prandtl number, E_c is Eckert number and S_c is Schmidt number.

The geometry of the multi-stenosis in dimensionless form is given by,

$$R(x) = \begin{cases} 1 & 0 \le x \le d_1 \\ 1 - \frac{\delta_1}{2}\left(1 + Cos \frac{2\pi}{l_1}\left(x - d_1 - \frac{l_1}{2}\right)\right) & d_1 \le x \le d_1 + l_1 \\ 1 & d_1 + l_1 \le x \le d_2 \\ 1 - \frac{\delta_2}{2}\left(1 + Cos \frac{2\pi}{l_2}\left(x - d_2 - \frac{l_2}{2}\right)\right) & d_2 \le x \le d_2 + l_2 \\ 1 & d_2 + l_2 \le x \le d_3 \\ 1 - \frac{\delta_3}{2}\left(1 + Cos \frac{2\pi}{l_3}\left(x - d_3 - \frac{l_3}{2}\right)\right) & d_3 \le x \le d_3 + l_3 \\ 1 & d_3 + l_3 \le x \le l \end{cases}$$

To solve equations (9) to (12) subject to the boundary conditions (13), forpurely oscillatory flow, let

$$\left.\begin{aligned} u(y,t) = u_f e^{i\omega t} \\ \theta(y,t) = \theta_f e^{i\omega t} \\ \varphi(y,t) = \varphi_f e^{i\omega t} \end{aligned}\right\} \qquad (14)$$

$$u(y,t) = (c_1 e^{m_1 y} + c_2 e^{-m_1 y} + c_3 e^{m_2 y} + c_4 e^{-m_2 y} + s_1)\ e^{i\omega t} \tag{15}$$

$$\theta(y,t) = (c_5 e^{m_3 y} + c_6 e^{m_4 y} + s_4)\ e^{i\omega t} \tag{16}$$

$$\varphi(y,t) = (c_7 e^{m_5 y} + c_8 e^{m_6 y})\ e^{i\omega t} \tag{17}$$

The blood vessels of the wall shear stress given by

$$\left(\frac{\partial u}{\partial y}\right)_{y=R} = c_1 m_1 e^{m_1 R} - c_2 m_1 e^{-m_1 R} + c_3 m_2 e^{m_2 R} - c_4 m_2 e^{-m_2 R} \tag{18}$$

Where the constant expressions are given Appendix

$$\frac{2a^2 e^{-i\omega t} K(1+m^2) n_2^2 \pi\ Re\ (Ps + P0\ \epsilon \cos[\omega\ t])}{(1+e^{n_1 R})(n_1^2 - n_2^2)(2\pi + 2m^2\pi + 2KM^2\pi + ia^2 K\alpha^2\omega + ia^2 Km^2\alpha^2\ \omega)}$$

$$c_2 = \frac{2a^2 e^{n_1 R - i\omega t} K(1+m^2) n_2^2 \pi\ Re\ (Ps + P0\ \epsilon \cos[\omega\ t])}{(1+e^{n_1 R})(n_1^2 - n_2^2)(2\pi + 2m^2\pi + 2KM^2\pi + ia^2 K\alpha^2\omega + ia^2 Km^2\alpha^2\ \omega)}$$

$$c_3 = -\frac{2a^2 e^{-i\omega t} K(1+m^2) n_1^2 \pi\ Re\ (Ps + P0\ \epsilon \cos[\omega\ t])}{(1+e^{n_2 R})(n_1^2 - n_2^2)(2\pi + 2m^2\pi + 2KM^2\pi + ia^2 K\alpha^2\omega + ia^2 Km^2\alpha^2\ \omega)}$$

$$c_4 = -\frac{2a^2 e^{n_2 R - i\omega t} K(1+m^2) n_1^2 \pi\ Re\ (Ps + P0\ \epsilon \cos[\omega\ t])}{(1+e^{n_2 R})(n_1^2 - n_2^2)(2\pi + 2m^2\pi + 2KM^2\pi + ia^2 K\alpha^2\omega + ia^2 Km^2\alpha^2\ \omega)}$$

$$n_1 = \sqrt{\frac{a^2 + \sqrt{a^4 - 4\left(\frac{i\omega\alpha^2 a^2}{2\pi} + \frac{M^2}{1+m^2} + \frac{1}{K}\right)}}{2}}$$

$$n_2 = \sqrt{\frac{a^2 - \sqrt{a^4 - 4\left(\frac{i\omega\alpha^2 a^2}{2\pi} + \frac{M^2}{1+m^2} + \frac{1}{K}\right)}}{2}}$$

$$n_3 = \sqrt{\frac{i\omega\ \mathrm{Pr}\ \alpha^2}{2\pi}}$$

$$n_4 = -\sqrt{\frac{i\omega\ \mathrm{Pr}\ \alpha^2}{2\pi}}$$

$$s_1 = \frac{Re\ a^2 (Ps + \epsilon\ P0 \cos[\omega t]) e^{-i\omega t}}{\frac{i\omega \alpha^2 a^2}{2\pi} + \frac{M^2}{1+m^2} + \frac{1}{K}}$$

$$s_2 = E_c \frac{(c_1 n_1 e^{n_1} - c_2 n_1 e^{-n_1} + c_3 n_2 e^{n_2} - c_4 n_2 e^{-n_2} - s_1)^2}{\frac{i\omega\alpha^2}{2\pi}}$$

$$s_3 = E_c \frac{(c_1 n_1 - c_2 n_1 + c_3 n_2 - c_4 n_2 - s_1)^2}{\frac{i\omega\alpha^2}{2\pi}}$$

$$c_5 = -s_3 - c_6$$

$$c_6 = \frac{1 - s_2 + s_3 e^{n_3}}{(e^{n_4} - e^{n_3})}$$

$$n_5 = \sqrt{\frac{i\omega\, \mathrm{S_c}\, \alpha^2}{2\pi} + K_c}$$

$$n_6 = -\sqrt{\frac{i\omega\, \mathrm{S_c}\, \alpha^2}{2\pi} + K_c}$$

$$c_7 = \frac{1}{(e^{n_5} - e^{n_6})}$$

$$c_8 = -\frac{1}{(e^{n_5} - e^{n_6})}$$

Result and Discussion

The numerical solutions of several parameters couple stress parameter (a), Hartmann number (M), Hall parameter (m), Reynolds number (Re), Prandtl number (Pr), Eckert number (Ec) are developed. It is mention that the axial field, temperature, shear stress and concentration are using physical parameters. The multi stenosis length $l_1 = l_2 = l_3 = 0.1$ to 0.3 , oscillatory flow $\omega = 1$, height of the multiple three stenosis of the blockage blood vessels are 10,20,30. $M = 1, 2, 3$, $m = 0.1$ to 1.0 , $Re = 1, 2, 3$, $Pr = 1, 3, 5$, $Ec = 1$ *to* 3 . The above physical values are numerically various values shows in Figure 2 to 11.

Figure 2 observed that axial field u decreases with increases of Reynolds number.

Figure 6 to 8 are the heat radiation of abnormal condition in blood fluid temperature. In Figure 6 temperature increases with increases of couple stress parameter. Figure 7 and 8 displays the temperature which decreases with increasing of prandtl number and Hall parameter. The cardiovascular system is part of the wall shear stress. Figure 9 depicts that

Figure 3 shows that impact of Magnetic parameter velocity, axial field which decreases while increasing values of M. Figure 4 blood fluid of the velocity decreases with couple stress parameter increases. Figure 5 shows that the velocity which decreases with increasing values of m.

variation of the upper wall shear stress which is decreases with increasing values of Hall parameter. Figure 10 demonstrate that concentration which reduces with increasing of chemical reaction. Figure 11 displays the concentration which increases with increasing of the schemidt number.

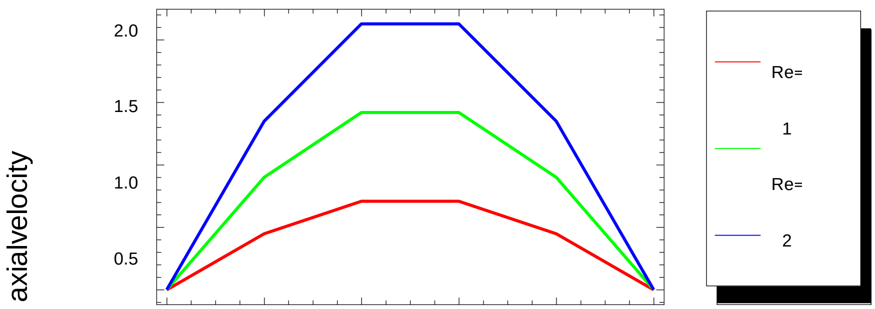


Fig. 2: Plots of velocity field with various parameters "Re"

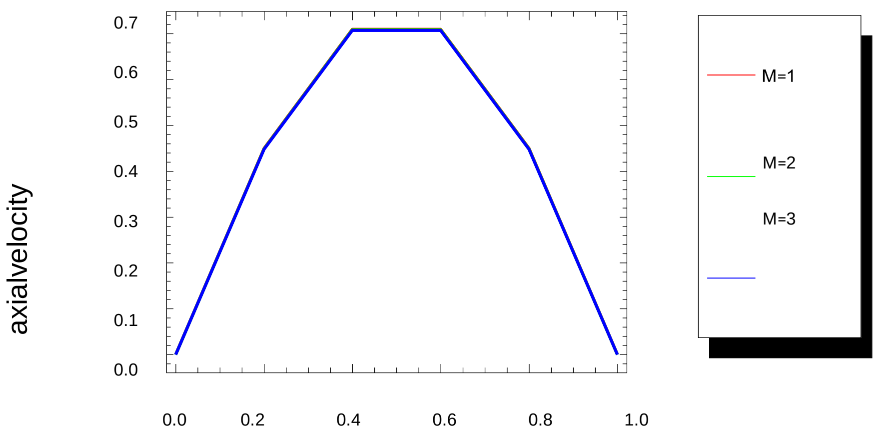
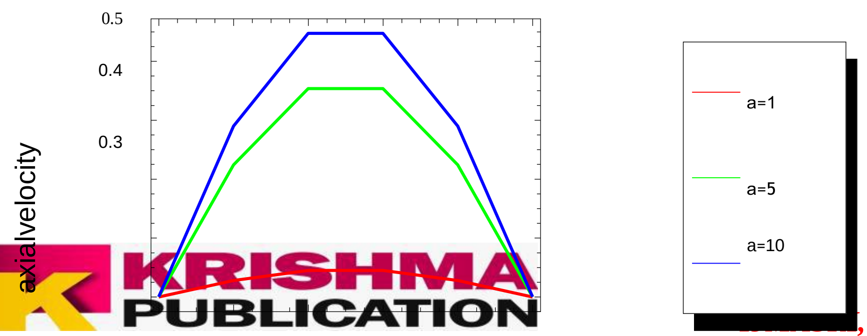


Fig. 3: Plots of velocity field with various parameter 'M'



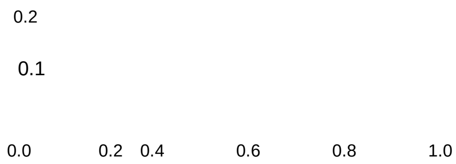


Fig. 4: Plots of velocity field with various parameter ”a”

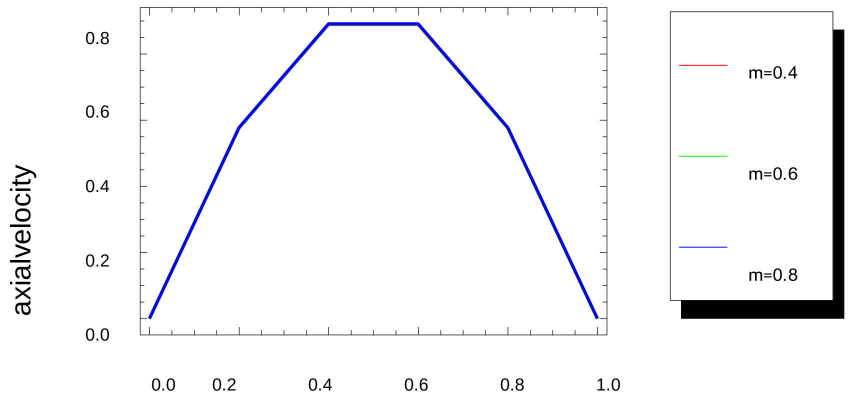


Fig.5: Plots of velocity field with various parameter ”m

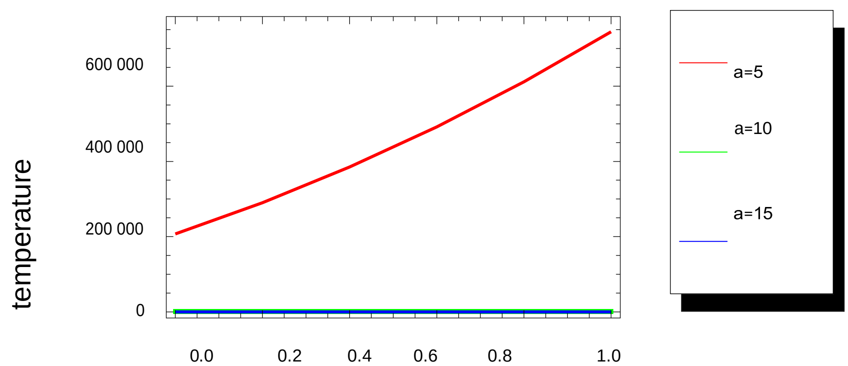


Fig. 6: Plots of temperature field with various parameter ”a”

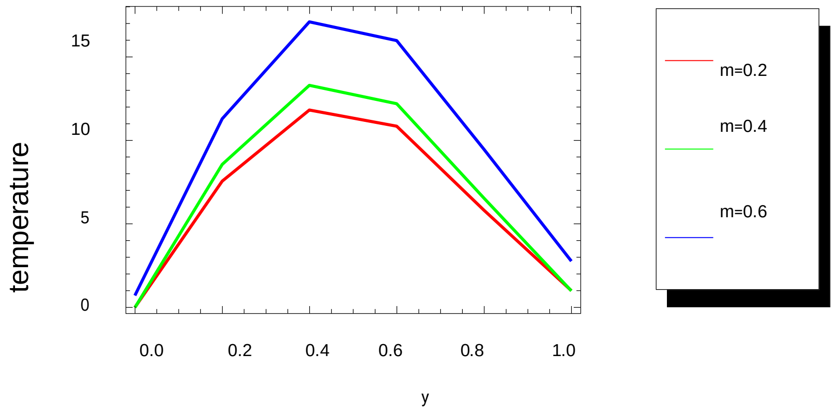


Fig. 7: Plots of temperature field with various parameter ”m

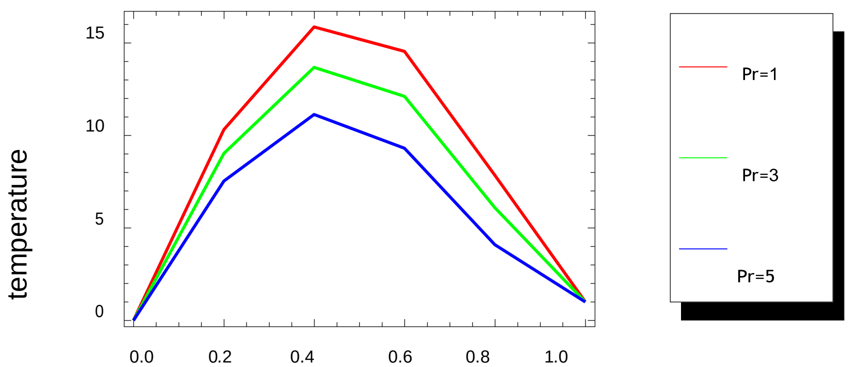


Fig. 8: Plots of temperature field with various parameter ”Pr”

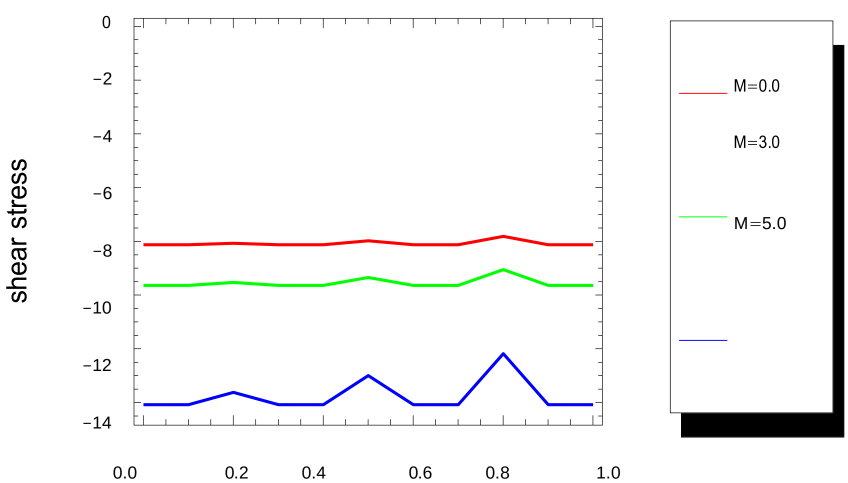


Fig. 9: Plots of wall shear stress field with various parameter”M”

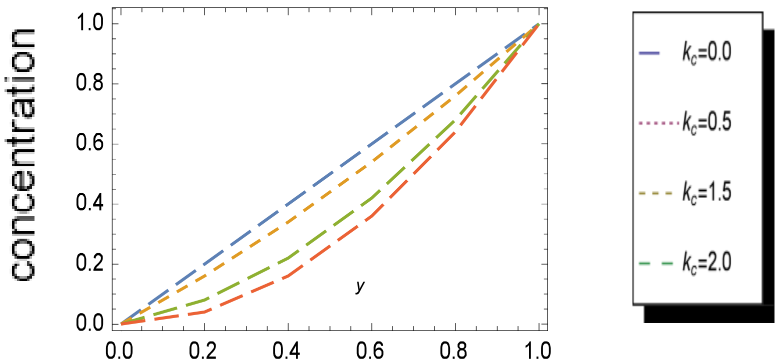


Fig. 10: Plots of concentration field with various parameter" K_c"

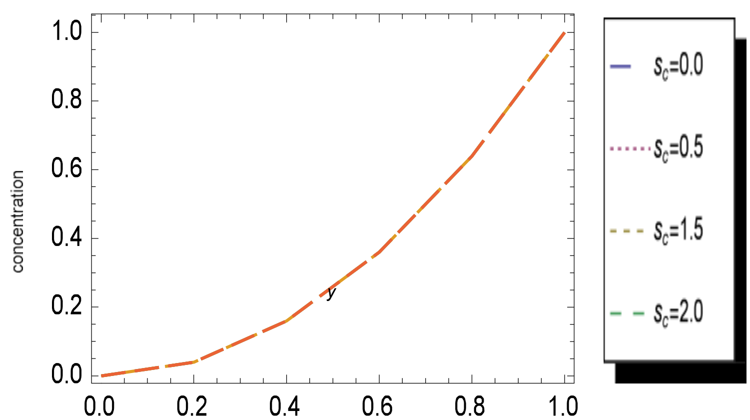


Fig.11: Plots of concentration field with various parameter "S_c"

Conclusion

The study of MHD with Hall current effect on oscillatory flow of blood with porous medium is investigated. The non-Newtonian blood fluid is assumed. This model will help to study the flow and concentration pattern of blood indiseased capillary.

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