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REVIEW ARTICLE

A STUDY ON VIRAL INFECTION MODEL WITH MODIFIED INFECTION RATE

[1]Sajad Ahmad Taray and [1]Tariq Ahmad Shikarie
[1]Department of Mathematics, Government Degree College Dooru Anantnag, Jammu & Kashmir

Abstract

In this chapter the main purpose is to study on viral infection model with modify infection rate. We perform a qualitative analysis and derive the All-Type dynamics which results from the appearance of bistable state or saddle-node state. In section 2.2, the qualitative analysis is performed, and the stability of the equilibria is obtained.

Key words: infection, infection equilibrium, bifurcation of cusp-type with codimension two

Introduction

Model Formation.

Let x(t) represents the number of susceptible cells which are produced at a constant rate λ die at a density-dependent rate dx, and became infected with a rate βuv, infected cells (y(t)) are produced at rate βuv and die at desity dependent rate ay: free virus particles v(t) are released from infected cells at the rate ky and die at a rate uv. The amount of free virus is simply proportional to the number of infected cells because the dynamics of virus is substantially faster than that of infected cells, u >> a, k>>λ. Thus the number of infected cells y(t) can also be considered as a major of virus load v(t) (Bernoulli, 1760)). The infection terms are based on the mass-action principle, that is the infection rate per susceptible cell and for virus is a constant β. However, infection experiments are of (Anderson and May 1998) suggests that the infection rate of microparasitic infections is an increasing function of the parasite dose and is usually sigmoidal in shape. Thus as (Anderson, R.M and May 1991), Regoes et. al and Yumei, *Yu*, Juan, J., Nieto and Kaifa Wang, Anderson, R.M and May 1991) take the non-linear

infection rate into account by relaxing the mass-action assumption. I have modified the infection rate of viral infection model of Yumai, Yu, Juan, J., Nieto and Kaifa Wang and the model is

$$\frac{dx}{dt} = \lambda - dx - \beta(y)x$$

$$\frac{dy}{dt} = \beta(y)x - ay$$

Here the infection rate per susceptible cell $\beta(y)$ is a sigmodial function of the virus (parasite) concentration because the number of infected cells y(t) can also be considered as a measure of virus load which is represented in the following form

$$\beta(y) = \frac{\left(\frac{y}{ID_{50}}\right)^k}{\rho + \left(\frac{y}{ID_{50}}\right)^k} \qquad \begin{matrix} k > 1 \\ \rho > 1 \end{matrix}$$

Here ID_{50} denotes theinfection dose at which 50% of the suspectible cells are infected. K measures the slope of the sigmodial curve at ID_{50}

$\left(\frac{y}{ID_{50}}\right)^k$ measures the infection force of the virus. $\frac{1}{\rho+\left(\frac{y}{ID_{50}}\right)^K}$ measures the inhibition effect from the behavioral change of the susceptible cells. Now take K = 2 is system (2.1.1) we have

$$\frac{dx}{dt} = \lambda - dx - \frac{\left(\frac{y}{ID_{50}}\right)^2}{\rho + \left(\frac{y}{ID_{50}}\right)^2} x$$

and $$\frac{dx}{dt} = \frac{\left(\frac{y}{ID_{50}}\right)^2}{\rho+\left(\frac{y}{ID_{50}}\right)^2} x - ay$$

and $$\frac{dx}{dt} = \lambda - dx - \frac{y^2}{\rho ID_{50}^2+y^2} x$$

(2.1.2)

and $$\frac{dy}{dt} = \frac{y^2}{\rho ID_{50}^2+y^2} x - ay$$

To be concise in notations, rescale (2.1.2) by $X=\frac{x}{ID_{50}}$ and $Y = \frac{y}{ID_{50}}$

For simplicity we still use variables x, y instead of X and Y and obtain

$$\frac{dXID_{50}}{dt} = \lambda - dXID_{50} - \frac{y^2 ID_{50}^2}{\rho ID_{50}^2 + Y^2 ID_{50}^2} XID_{50}$$

$$\frac{dXID_{50}}{dt} = \lambda - dXID_{50} - \left(\frac{y^2}{\rho + Y^2}\right) XID_{50}$$

multiply both sides by $\frac{1}{ID_{50}}$ we get

$$= \frac{1}{ID_{50}}, \frac{dXID_{50}}{dt} = \frac{\lambda}{ID_{50}} - \frac{dXID_{50}}{ID_{50}} - \frac{1}{ID_{50}}\left[\frac{y^2}{\rho + Y^2}\right] XID_{50}$$

$$= \frac{dx}{dt} = m - dx - \left[\frac{y^2}{\rho + Y^2}\right] x \qquad where \qquad m\frac{\lambda}{ID_{50}}$$

Or $$\frac{dx}{dt} = m - dx - \frac{y^2}{\rho+y^2} x$$

$$\frac{dy}{dt} = \frac{y^2}{\rho + y^2} x - ay$$

Where m=$\frac{\lambda}{ID_{50}}$ note that $\frac{1}{d}$ is the average life time of susceptible cells and $\frac{1}{a}$ is the average life time infected cells. Thus a ≥ d is always valid by means of biological detection. If a = d the virus does not kill infected cells. Therefore the virus is non cytophatic in vivo. However when a>d which means the virus kills infected cells before its average life time. The is virus is cytophatic in vivo.

Qualitative Analysis

In this section our main aim is to study the effect of the non-linear infection rate on the dynamics of (2.1.3) we will perform a qualitative analysis and derive the allee type dynamic which results from the appearance of bistable states or saddle node in (2.1.3). In this section the qualitative analysis of (2.1.3) is performed, and the stability of the equilibria is obtained.

In order to find positive (infection) equilibria we set

$$m - dx - \frac{y^2}{p+y^2}x = 0 \quad (2.2.1)$$

And $$\frac{y^2}{p+y^2}x = ay = o$$

We have from equation (2.2.2), we have

$$y\left(\frac{y}{\rho + y^2}x\right) = ay$$

Based on this equation, we have

(i) There is no infection equilibria if $m^2 < 4a^2d\rho(1+d)$

(ii) There is unique infection equilibrium $E_1 = (x^*, y^*)$ if $m^2 = 4a^2d\rho(1+d)$

(iii) There are two infection equilibria $E_{12} = (\overline{x_2}, \overline{y_2})$ of $m^2 > 4a^2d\rho(1+d)$

Since equation (2.2.3) is quadratic in y

∴ we have

$$= \frac{m \pm \sqrt{m^3 - 4a^2d\rho(1+d)}}{2a(1+d)}$$

thus $y^* = \frac{m}{2a(1+d)}$ $\qquad x^* = a\frac{(\rho+y^{*2})}{y^*}$

$$\overline{y}_1 = \frac{m-\sqrt{m^2-4a^2d\rho(1+d)}}{2a(1+d)} \qquad \overline{x}_1 = \frac{a\left(\rho+\overline{y}_1^2\right)}{y^*}$$

$$\overline{y}_2 = \frac{m+\sqrt{m^2-4a^2d\rho(1+d)}}{2a(1+d)} \qquad \overline{x}_1 = \frac{a\left(\rho+\overline{y}_1^2\right)}{y^*}$$

Thus the surface SN = {(m, d, a): $m^2 = 4a^2d\rho(1+d)$} (2.2.5) is a saddle – node – bifurcation surface, that is on one side of the surface SN system (2.1.3) has not any positive equilibria on the surface SN system (2.1.3) has only one positive equilibria: on the other side of the surface SN System (2.1.3) has two positive equilibria. The detailed results will follow now we determine the stability of E_{11} and E_{12}, for that we proceed as below,

let $$F_1 = m - dx - \frac{y^2}{\rho+y^2}x$$

and $$F_1 = \frac{y^2}{\rho+y^2}x - ay$$

now $$\frac{\partial F_1}{\partial x} = -d - \frac{y^2}{\rho+y^2}$$

$$\frac{\partial F_1}{\partial y} = \frac{[(\rho+y^2)2y - y^2(2y)]x}{(\rho+y^2)^2}$$

$$= -\frac{[2y\rho+2y^3-2y^3]x}{(\rho+y^2)^2} = \frac{2ypx}{(p+y^2)^2}$$

Also $$\frac{\partial F_2}{\partial x} = \frac{y^2}{\rho+y^2}$$

And $$\frac{\partial F_2}{\partial y} = -\frac{[(\rho+y^2)2y-y^2(2y)]x}{(\rho+y^2)^2} - a$$

$$= -\frac{[2y\rho+2y^3-2y^3]x}{(\rho+y^2)^2} - a$$

$$= \frac{2y\rho x}{(\rho+y^2)^2} - a$$

the Jacobi matrix at $E_{11}$is

$$J_{E_{11}} = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{bmatrix}$$

$$=\frac{a(1+d)\left[4a^2d\rho(1+d)+m\left(\sqrt{m^2-4a^2d\rho(1+d)}-m\right)\right]}{2a^2\rho(1+d)+m\left(m-\sqrt{m^2-4a^2d\,\rho(1+d)}\right)}$$

Since $m^2 > 4a^2d\rho(1+d)$ in this case,
$4a^2d\rho(1+d)+m\left(\sqrt{m^2-4a^2d\rho(1+d)}-m\right) > 0$ *is valid*
Thus $\det(J_{E_{11}}) < 0$ and the equilibrium E_{11} *is saddle.*
Now we determine the stability of E_{12} for that we have the Jacobian matrix at E_{12} *is*

$$J_{E_{12}} = \begin{bmatrix} -d - \frac{\overline{y}_2^2}{\rho+\overline{y}_2^2} & -\frac{2\overline{x}_2\overline{y}_2\rho}{(\rho+\overline{y}_2^2)^2} \\ \frac{\overline{y}_2^2}{\rho+\overline{y}_2^2} & -a + \frac{2\overline{x}_2\overline{y}_2\rho}{(\rho+\overline{y}_2^2)^2} \end{bmatrix}$$

$$\therefore \det(J_{E_{12}}) = \frac{\left(-d\left(\rho+\overline{y}_2^2\right)-\overline{y}_2^2\right)\left(-a\left(\rho+\overline{y}_2^2\right)^2+2\overline{x}_2\overline{y}_2\,\rho\right)+2\rho\overline{y}_2^3x_2}{\left(\rho+\overline{y}_2^2\right)^3}$$

$$= \frac{a(1+d)\left[-4a^2d\rho(1+d)+m\left(\sqrt{m^2-4a^2d\rho(1+d)}+,\ m\right)\right]}{2a^2\rho(1+d)+m\left(m+\sqrt{m^2-4a^2d\rho(1+d)}\right)}$$

Det $(J_{E_{12}})$>0 thus the equilibrium $E_{12}$is a node, or a focus, or a center.
For the sake of simplicity, we denote

$m_\in = 2a\sqrt{d(1+d)}, m_\circ = \frac{a^2(1+2d)}{\sqrt{(a-d)(1+a+d)}}$ *if* $a > 2d(1+d)$

on the stability of E_{12}we have the following results:

Theoreum 2.1. Suppose that equilibrium F_{12} exists, that is $m>m_\in$.

Then E_{12} is always stable is $d \le a \le 2d(1+d)$. When $a> 2d(1+d)$, we have

(i) E_{12} is stable if $m > m_\circ$
(ii) E_{12} is unstable if $m > m_\circ$
(iii) E_{12} is linear center if $m > m_\circ$

Proof :we have the Jacobian matrix of E_{12} as

$$J_{E_{12}} = \begin{bmatrix} \frac{-d\left(\rho+\overline{y}_2^2\right)-\overline{y}_2^2}{\left(\rho+\overline{y}_2^2\right)} & \frac{-2\overline{x}_2\overline{y}_2\rho}{(\rho+\overline{y}_2^2)^2} \\ \frac{\overline{y}_2^2}{\rho+\overline{y}_2^2} & \frac{-a(\rho+\overline{y}_2^2)^2+2\overline{x}_2\overline{y}_2\rho}{(\rho+\overline{y}_2^2)^2} \end{bmatrix}$$

Now

$$\mathrm{tr}(J_{E_{12}}) = \frac{-d\left(\rho+\overline{y}_2^2\right)-\overline{y}_2^2}{\left(\rho+\overline{y}_2^2\right)^3} + \frac{\left(-a\left(\rho+\overline{y}_2^2\right)^2\right)+2\overline{x}_2\overline{y}_2\,\rho+}{\left(\rho+\overline{y}_2^2\right)^2}$$

Thus

$$\mathrm{tr}(J_{E_{12}}) = \frac{2a^2\rho(1+d)(1+2d)-m(1+a+d)\left[m+\sqrt{m^2-4a^2d\rho(1+d)}\right]}{2a^2(1+d)p+m\left[m+\sqrt{m^2-4a^2d\rho(1+d)}\right]}$$

and its sign is determined by

$$F(m) = 2a^3{}_\rho(1+d)(1+2d) - m(1+a+d)\left[m+\sqrt{m^2-4a^2d_\rho(1+d)}\right] \quad (2.2.8)$$

$$F(m) = -(1+a+d)\left[2m+\sqrt{m^2-4a^2d\rho(1+d)} + \frac{m^2}{\sqrt{m^2-4a^2d\rho(1+d)}}\right]$$

(2.2.9)

Which means that F (m) is a monotone decreasing function of variable m

Note that F(m)=0 implies that

$$\frac{2a^3\rho(1+d)(1+2d)}{-m(1+a+d)} - m = \sqrt{m^2 - 4a^2d\rho(1+d)}$$

(2.2.10)

This means that $F(m_\circ) = 0$ thus under the condition of m>$m_\in$ and sign of F(m), tr($J_{E_{12}}$) <0 is always valid if a ≤ 2d (1+d), when a>2d(1+d), tr($J_{E_{12}}$) < $0\ if\ m > m_\circ$, and tr($J_{E_{12}}$) $= 0\ if\ m = m$

Theorem 2.2. There is no limit cycle in (2.1.3) if either of the following condition hold:

(i) a=d and $m^2 > 4a^2d\rho(1+d)$

(ii) $d < a < 2d(1+d) and\ 4a^2d(1+d)\rho < m^2 < \frac{a^4\rho(1+2d)^2}{(a-d)(1+a+d)}$

When $m^2 = 4a^2d(1+d)\rho$, system (2.1.3) has a unique infection equilibrium E_1. The Jacobian matri at E_1 is

$$J_{E_{12}} = \begin{bmatrix} -d\frac{-y^{*2}}{\rho+y^{*2}} & \frac{2x^*y^*\rho}{(\rho+y^{*2})} \\ \frac{y^{*2}}{(\rho+y^{*2})} & -a+\frac{2x^*y^*\rho}{(\rho+y^{*2})^2} \end{bmatrix}$$

Thus tr $(J_{E_1}) = \frac{4a^2(1+d)\rho[a-2d(1+d)]}{4a^2(1+d)^2+p+m^2}$

Hence E_1 is a degenerate singular point
Hence the theorem is proved

Conclusion

Modeling result is helpful to predict the developing tendency of disease. The model we have discussed provides learning about the effect of non-linear infection rate, and we also study the stability of infection equilibria. As we have found that when $\det(J_{E_{11}}) < 0$ the equilibrium E_{11} is a saddle and when $\det(J_{E_{11}}) > 0$ the equilibrium E_{12} is a node, o a focus or a centre. Also we have observed that (i) when $m > m_\circ, then\ E_{12}$ is stable (ii) when $m < m_\circ$ then E_{12} in unstable and (iii) when $m = m_\circ$ then E_{12} is a linear centre.

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